ISI – Bangalore Center – B Math - Physics II – End Semestral Exam
Date: 9 November 2016. Duration of Exam: 3 hours
Total marks: 50

ANSWER ALL QUESTIONS

Q 1 [Total Marks: 3+1+2+4]

Let $P(V-b)=RT$ be the equation of state for one mole of a certain gas, where b is a constant . Show that
a.) the molar internal energy u is a function of temperature only,
b.) c_v depends only on temperature,
c.) $c_p - c_v = R$.
c.) Assuming further c_v is a constant show that in an adiabatic quasistatic process in which (P_f, V_f), (P_i, V_i) are the final and the initial states, the work done by the gas is given by $\dfrac{P_f V_f - P_i V_i - b(V_f - V_i)}{1-\gamma}$ where $\dfrac{c_p}{c_v} = \gamma$.
[HINT: Use one of the "$T\,dS$" equations and Maxwell relations, as needed]

Q2. [Total Marks:6+4=10]

a.) Show that for an engine working between reservoirs at different temperatures, $\sum \dfrac{Q_i}{T_i} \le 0$ where Q_i is the amount of heat exchanged with the reservoir at temperature T_i. Note that Q_i is positive if heat is absorbed and negative if heat is given up.

b.) Suppose an engine is working between various heat reservoirs operating at different temperatures. Let $T_{\max}$ be the maximum temperature of those heat reservoirs from which the engine is receiving heat and let $T_{\min}$ be the minimum temperature of those heat reservoirs to which the engine is releasing heat. Assume that $T_{\min} < T_{\max}$. Show that the efficiency of the engine cannot exceed the efficiency of an Carnot engine operating between $T_{\max}$ and $T_{\min}$.

Q3. [Total Marks:2+3+1+4]

A perfect crystal has N lattice sites and M interstitial locations. An atom can be present either at a lattice site or an interstitial location, however, an amount of energy $\Delta(\Delta > 0)$ is required to move an atom from a lattice site to an interstitial site. The crystal is in contact with a heat reservoir at temp T. Suppose that the total number of atoms is N.

a.) Calculate the number of possible states if the number of atoms that have been placed in interstitial sites is n. [Check your answer for correctness by setting n=1.]

b.) Suppose there are n atoms in the interstitial sites already. Due to thermal fluctuations, one atom leaves the interstitial site and return to the lattice site. Show that the change of entropy of the crystal is exactly given by

$$k \ln \frac{n^2}{(N-n+1)(M-n+1)}.$$

c.) Calculate the change of entropy of the reservoir for this transition.

d.) Using b.) and c.) above show that when the crystal is in thermal equilibrium, then in the limit N>>1. M >> 1,

$$n = \sqrt{NM}\, e^{-\frac{\Delta}{2kT}} \text{ for } kT \ll \Delta \text{, and } \; n = \frac{NM}{N+M} \text{ for } kT \gg \Delta$$

[You can use the result in b.) for part d.) even if you cannot derive the result in b.)]

Q 4. [Total Marks: 1+5+4=10]

a.) Define the partition function for a system in contact with a reservoir at temperature T.

b.) State the equipartition theorem and prove it using the partition function.

c.) Using the equipartition theorem, calculate the specific heat capacity of a diatomic gas who molecules behave like a rigid rotor [the two atoms connected by a rigid rod] AND also for a diatomic gas whose molecules behave like two atoms connected by a standard spring. Your answer must identify the relevant parameters contributing to the specific heats.

Q5.[Total Marks:4+4+2=10]

a.) White light is incident normally on a thin film suspended in air. The refractive index of the thin film is 1.5 and its thickness is 5000 Å. For what wavelengths in the visible spectrum (4000 Å to 7000 Å) will the intensity of the reflected light be a maximum?

b.) Suppose we have two coherent sources placed at two slits distance *d* apart. The amplitude of the wave (Electric field) at one of the slits is twice the amplitude at the other slit. Show that on a screen placed at a distance large compared to *d*, the interference pattern in terms of intensities is given by

$$I = I_0[5 + 4\cos(\frac{2\pi d \sin\theta}{\lambda})]$$

where I_0 is the intensity of light due to the source with lower intensity, and θ is the angle between the line connecting a point on the screen to the mid point between the slits and the perpendicular drawn at the mid point (in the usual notation, see Figure 1.) .

c.) How is the result in part b.) is to be modified if each of the slits is taken to have finite width *b*?

Maxwell Relations that you may use or not use:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

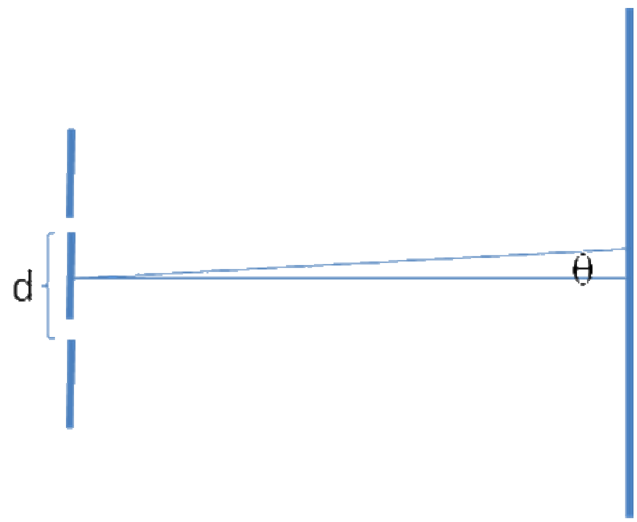

Figure 1. Not drawn to scale. The distance to the screen is large compared to *d*.